Treatment Effect and Program Evaluation — Identification of Causal Effect Using Instrumental Variables*

Usual treatment of causal effect: Use structural equation(s) model. Sometime the estimation is assisted by instrumental variables (IV)

Alternative approach (mostly in statistics): Use randomize experiment

Basic notion: *potential outcome*. That is, the effect is the difference between the value if the unit of observation is treated and the value if the unit is not treated.

The target of investigation: Average causal effect, the Rubin's causal model (RCM).

Key result: There is a link between the two approaches, i.e., randomize treatment and instrumental variables. Having an IV is as if a research was able to randomize people.

You observe a group of peopel who do not serve the army. and then you also observe a group of people who serve the army. Comparing the outcomes of two groups doesn't give us anything UNLESS people who serve in the army or not is random.

Application: The effect of military service on health outcome.

Notation:

Z = Draft status ($Z = 1$ if individual was drafted; $Z = 0$ otherwise).

D = Treatment. ($D = 1$ if individual served in the military, $D = 0$ otherwise)

With perfect compliance it should be that $Z = D$. Nevertheless, this is rarely the case.

Y = the outcome variable, i.e., the health status

Remark: Extension to the binary case and to cases with covariates are straightforward. The latter can be easily achieved by applying the mothed (discussed below) to distinct values of the covariates.

*By Joshua D. Angrist, Guido W. Imbens, and Donald B. Rubin, Journal of the American Statistical Association, Vol. 91, No. 434 (Jun., 1996), pp. 444-455

1 Structural Equation Model

An equation represents the causal link through a dummy endogenous model, i.e., the effect of D_i on Y_i:

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i$$

where

$$D_i = \begin{cases} 1 & \text{if } D_i^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

and the latent variables D_i^* is given by:

$$D_i^* = \alpha_0 + \alpha_1 Z_i + \upsilon_i$$

where here, β_1 is the *causal effect* of D on Y. The problem arises from the fact that D_i is a choice variable. Therefore, we have reason to believe that D_i is correlated with ε_i.

It is endogenous due to the potential correlation between ε and υ.

Z_i -> D_i^* -> $D_i - > Yi$

Assumption 1 (for the identification of β_1)

$$E\left(Z\varepsilon\right) = 0 \text{ and } E\left(Z\upsilon\right) = 0$$

i.e., Z is uncorrelated with both errors. Note that the variable Z does not have any direct effect on Y, the effect of Z of Y is *only* through its effect on D. Z affects Y only through D.

Z is exogenous and in not correlated with any errors.

Assumption 2

$$Cov\left(D, Z\right) \neq 0.$$

Under Assumptions 1 and 2 Z is considered to be an IV.

An IV estimator:

$$\widehat{\beta}^{IV} = \frac{\widehat{Cov}\left(Y, Z\right)}{\widehat{Cov}\left(D, Z\right)} = \frac{\frac{\sum_{i=1}^{N} Y_i Z_i}{\sum_{i=1}^{N} Z_i} - \frac{\sum_{i=1}^{N} Y_i (1-Z_i)}{\sum_{i=1}^{N} (1-Z_i)}}{\frac{\sum_{i=1}^{N} D_i Z_i}{\sum_{i=1}^{N} Z_i} - \frac{\sum_{i=1}^{N} D_i (1-Z_i)}{\sum_{i=1}^{N} (1-Z_i)}}$$

where the second equality follows from the binary nature of the instrument. This model is generally ignored by statisticians due to the critical assumptions made, especially regarding ε and υ.

2 Causal Estimation with Instrumental Variables

Rubin Causal Model (RCM):

Z stays the same as before. However, the variables Y and D change, because of the necessity to deal with *potential outcomes.*

Notation:

$Z = N$-dimensional vector of assignment with the i-th element Z_i. $Z = (z_1, z_2, ...z_n)'$

$D_i(Z)$ and $Y_i(Z, D)$ are the *potential outcomes.* These outcomes are only partially revealed by the assignment mechanism.

Initail goal: Provide inference on the finite population of N units under study, to which Z is randomly assigned. Some assumptions are needed about the degree of interference between units.

Assumption 1 —— Stable Unit Treatment Value Assumption (SUTVA):

(a). if $Z_i = Z_i'$ then $D_i(Z) = D_i(Z')$; and

(b). if $Z_i = Z_i'$ and $D_i = D_i'$ then $Y_i(Z, D) = Y_i(Z', D')$.

This assumption implies that the units are uncorrelated and are unaffected by the treatment status of others (this is almost an i.i.d assumption). Therefore

$$Y_i(Z, D) = Y_i(Z_i, D_i) \text{ and } D_i(Z) = D_i(Z_i).$$

Definition 1 —–*Causal effect of Z on D and of Z on Y:*

The causal effect for individual: $D_i(1) - D_i(0)$.(Intent to treat effect on D)

The causal effect of Z on Y is: $Y_i(1, D_i(1)) - Y_i(0, D_i(0))$.(Intent to treat effect on Z)

These are the intention-to-treat effects, where there is imperfect compliance with the assigned treatment.

Assumption 2 ——Random assignment (RA):

Z_i is randomly assigned.

The treatment assignment Z_i is random. That is,

$$\Pr(Z = C) = \Pr(Z = C'),$$

for all C and C' such that $l^T C = l^T C'$.

The use of the assumptions: Given the SUTVA and RA assumptions, classify Y and D by the values of Z and take differences to get an unbiased estimator of the intension-to-treat effect. That is,

$$\begin{aligned}
\frac{\sum_{i=1}^{N} Y_i Z_i}{\sum_{i=1}^{N} Z_i} - \frac{\sum_{i=1}^{N} Y_i (1 - Z_i)}{\sum_{i=1}^{N} (1 - Z_i)} &= \frac{\sum_{i=1}^{N} Y_i Z_i \sum_{i=1}^{N} (1 - Z_i) - \sum_{i=1}^{N} Y_i (1 - Z_i) \sum_{i=1}^{N} Z_i}{\sum_{i=1}^{N} Z_i \sum_{i=1}^{N} (1 - Z_i)} \\
&= \frac{\frac{1}{N} \sum_{i=1}^{N} Y_i Z_i - \left(\frac{1}{N} \sum_{i=1}^{N} Y_i\right) \left(\frac{1}{N} \sum_{i=1}^{N} Z_i\right)}{\frac{1}{N} \sum_{i=1}^{N} Z_i Z_i - \left(\frac{1}{N} \sum_{i=1}^{N} Z_i\right) \left(\frac{1}{N} \sum_{i=1}^{N} Z_i\right)}.
\end{aligned}$$

and similarly for the intention-to-treat effect of Z on D.

The ratio of the two estimates give the IV estimate defined above in equation (1).

Instrumental Variables:

Since compliance is not perfect, the above estimator is not a consistent estimator of the effect of D on Y. For this we need some additional assumptions:

Assumption 3 —–Exclusion Restrictions:

$$Y(Z, D) = Y(Z', D)$$

for all Z, Z' and for all D. Only D influences your (health) outcomes. Z doesn't affect your (health) outcomes. That is, Z or Z' are irrelevant.

This assumption states that the treatment assignment is unrelated to potential outcomes, once the treatment received is taken into account. This implies that

$$Y_i(1, d) = Y_i(0, d) \quad \text{for } d = 0, 1.$$

This is the equivalent to the assumption on Z considered before in the structural equation formulation. The real problem is that $Y_i(1, d)$ and $Y_i(0, d)$ are not jointly observed.

This assumption is very crutial but not testable.

Remark: Note that since $Y_i(1, d)$ and $Y_i(0, d)$ cannot be jointly observed, Assumption 3 is not verifiable.

By Asssumption 3 we have that

$$Y(D) = Y(Z, D) = Y(Z', D) \quad \text{for all } Z, \text{Z}' \text{ and for all } D.$$

By Assumption 1 we have that

$$Y_i(D_i) = Y_i(Z, D).$$

Definition 2 —–*Causal effect of Z on D:*

The causal effect of D on Y, for the i-th person, is given by:

$$Y_i(1) - Y_i(0).$$

Note that there is an inherent problem in this definition, since we can never observe both $Y_i(1)$ and $Y_i(0)$.

$Y_i(1) - Y_i(0)$ is what I am after.

Notation:

$E(g) =$ mean over the N units of any function $g(\cdot)$ of $Z_i, D_i(1), D_i(0), Y_i(0,0), Y_i(0,1), Y_i(1,0)$ or $Y_i(1,1)$.

$E(g|h(\cdot) = h(0)) =$ mean over a subpopulation for which $h(\cdot) = h_0$.

Relative size of a subpopulation with $h(\cdot) = h_0$:

$$\Pr(h(\cdot) = h_0) = E(1(h(\cdot) = h_0)).$$

Assumption 4:

Non-zero average causal treatment effect of Z on D:

$$E(D_i(1) - D_i(1)) \neq 0.$$

The interpretation of this assumption is that Z has some effect on the average probability of treatment.

On average, people who are drafted are more likely to serve the army.

Assumption 5 ——Monotonicity:

$$D_i(1) \geq D_i(0) \quad \text{for all } i = 1,...,N.$$

Monotonicity assumption is used to kill defiers.

Definition 3 *—Instrumental variable for the causal effect of D on Y:*

A variable Z is an instrumental variable for the causal effect of D on Y if:

1. its average effect on D is non-zero;
2. it satisfies the exclusion $(Y(D) = Y(Z, D))$and monotonicity assumption (If you are drafted, you are more likely to serve);
3. it is randomly assigned; and
4. SUTVA holds.

3 Interprepation of tht Instrumental Variable Estimation

SUTVA and the exclusion restriction allows one to establish the relation between intention-to-treat effects of Z on Y and D, and the causal effect of D on Y at the individual level:

$$\begin{aligned}
& Y_i(1, D_i(1)) - Y_i(0, D_i(0)) \\
= \ & Y_i(D_i(1)) - Y_i(D_i(0)) \\
= \ & [Y_i(1) Y_i(1) D_i(1) + Y_i(0)(1 - D_i(1))] \\
& - [Y_i(1) D_i(0) + Y_i(0)(1 - D_i(0))] \\
= \ & (Y_i(1) - Y_i(0))(D_i(1) - D_i(0))
\end{aligned}$$

Therefore

$$\begin{aligned}
& E[Y_i(1, D_i(1)) - Y_i(0, D_i(0))] \\
= \ & E[(Y_i(1) - Y_i(0))(D_i(1) - D_i(0))] \\
= \ & E[(Y_i(1) - Y_i(0)) | D_i(1) - D_i(0) = 1] \Pr(D_i(1) - D_i(0) = 1) \\
& - E[(Y_i(1) - Y_i(0)) | D_i(1) - D_i(0) = -1] \Pr(D_i(1) - D_i(0) = -1)
\end{aligned}$$

Note that the weights add up to $\Pr(D_i(1) \neq D_i(0))$.

With monotonicity (i.e., $D_i(1) - D_i(0) = 1$, or $D_i(1) - D_i(0) = 0$) we have

$$\begin{aligned} & E[Y_i(1, D_i(1)) - Y_i(0, D_i(0))] \\ = \ & E[(Y_i(1) - Y_i(0)) | D_i(1) - D_i(0) = 1] \Pr(D_i(1) - D_i(0) = 1). \end{aligned}$$

Proposition 4 —–*Causal interpretation of IV estimator:*

Given Assumptions 1, 3, 4 and 5, the instrumental variable estimator is:

$$\frac{E[Y_i(1, D_i(1)) - Y_i(0, D_i(0))]}{E[D_i(1) - D_i(0)]} = E[(Y_i(1) - Y_i(0)) | D_i(1) - D_i(0) = 1], \tag{1}$$

$$E[(Y_i(1) - Y_i(0)) | D_i(1) - D_i(0) = 1] = \frac{E[Y_i(1, D_i(1)) - Y_i(0, D_i(0))]}{E[D_i(1) - D_i(0)]} = \text{ IV estimator using } (1,Z_i) \text{ as IV of } (1,D_i)$$

which is the local average treatment effect (LATE).

Table 1. Causal Effect of Z on Y, $Y_i(1, D_i(1)) - Y_i(0, D_i(0))$, for the Population Units Classified by $D_i(10)$ and $D_i(0)$.

		$D_i(0)$	
		0	1
$D_i(1)$	0	$Y_i(1,0) - Y_i(1,0) = 0$ **Group A:** Never-taker	$Y_i(1,0) - Y_i(1,0) = -(Y_i(1) - Y_i(0))$ **Group B:** Defier
	1	$Y_i(1,1) - Y_i(0,0) = Y_i(1) - Y_i(0)$ **Group C:** Complier	$Y_i(1,1) - Y_i(0,1) = 0$ **Group D:** Always-taker

1. **Group C:** $D_i(1) - D_i(0) = 1$. There are the individuals who are induced to take the treatment by the assignment to the treatment. There are compliancers for whom the causal treatment effect is $Y_i(1) - Y_i(0)$. Group C is not the entire populatio, that is why it is called LOCAL average treatment effect.

2. **Group A:** $D_i(1) - D_i(0) = 0$. There are the individuals who always avoid the draft, regardless of the assignment.

3. **Group D:** $D_i(1) - D_i(0) = 0$. There are the individuals who always serve, regardless of the assignment.

   In these last two groups the individuals do not change the treatment status by the assignment. Therefore, the effect of Z on Y is zero by the exclusion restriction.

4. **Group B:** $D_i(1) - D_i(0) = -1$. There are the individuals who do the opposite of their assignment.

The groups of A, B and D (together) are the non-compliers. By virtue of the assumption we have that:

(i). Group A and D have zero effect of Z on Y; and

(ii). Group B dose not exist.

(iii). By virtue of Assumption 4, the subpopulation of compliers is greater than zero, and the effect equals to the average causal effect of Z on D.

Overall, the assumption imply that the average causal effect of Z on Y is proportional to the average causal effect of D on Y, for the group of compliers. (This is what Proporsition 1 establishes.)

Remark: Under the assumptions made above one can not generally identify the specific members of the group of compliers, defined by $D_i(0) = 0$ and $D_i(1) = 1$, for whom we can identify the average treatment effect.

Consequently, LATE is not ATE for either the entire population, not any subpopulation identifiable from observed values. To achieve this, some more assumption are needed, some of which are not verifiable.

Note that random assignment of Z_i does not garantee that exclusion restriction is satisfied. Random assignment implies that Z_i is independent of D_i. But it does not imply that Z_i is independent of $Y_i(1)$ and $Y_i(0)$.