Lecture 3: Models of Self-Selection (continued) —— Comparative Advantage

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1 Math Review (Properties of Truncated Normal Random Variable)

(P-1) $E(Z|Z>c) = \frac{\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{c^2}{2}\right)}{\Phi(-c)} = \frac{\phi(c)}{\Phi(-c)} \equiv \lambda(c)\,;\ \lambda(c) \geq 0$ and $\lambda(c) \geq c$

(P-2) $Var(Z|Z>c) = 1 + \lambda(c)\,c - (\lambda(c))^2$
$= 1 + \lambda(c)(c - \lambda(c))$
$= 1 - E(Z|Z>c)\,E(Z-c|Z>c)$

(P-3) $\underset{c\to\infty}{Lim}\lambda(c) = \infty$

(P-4) $\underset{c\to-\infty}{Lim}\,\lambda(c) = 0$

(P-5) $0 < \frac{\partial\lambda(c)}{\partial c} = \lambda'(c) = \lambda(c)(\lambda(c) - c) < 1$ for $-\infty < c < \infty$

(P-6) $\underset{c\to-\infty}{Lim}\,\frac{\partial\lambda(c)}{\partial c} = 0$

(P-7) $\underset{c\to\infty}{Lim}\frac{\partial\lambda(c)}{\partial c} = 1$

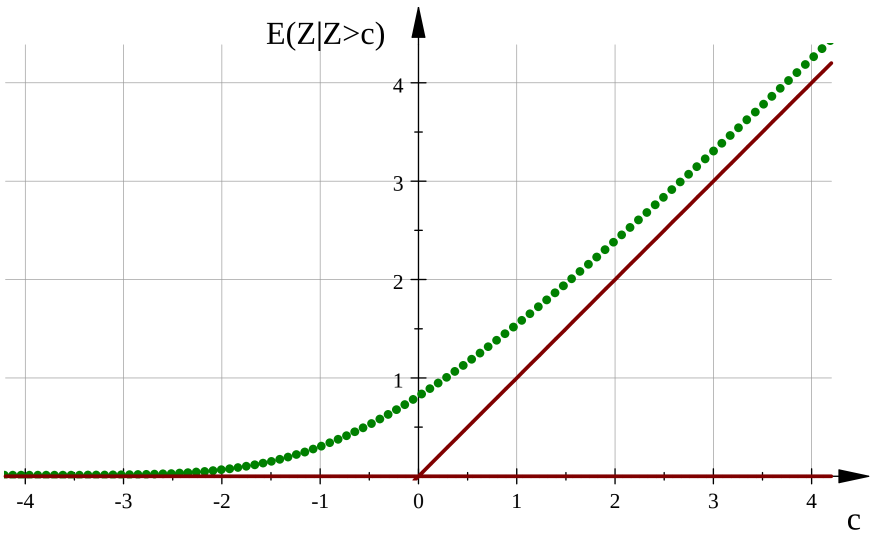
(P-8) $\frac{\partial^2\lambda(c)}{\partial c^2} > 0$ for $c < \infty$

(P-9) $\frac{\partial Var(Z|Z>c)}{\partial c} < 0$ for $c < \infty$

(P-10) $\underset{c\to\infty}{Lim}Var(Z|Z>c) = 0$

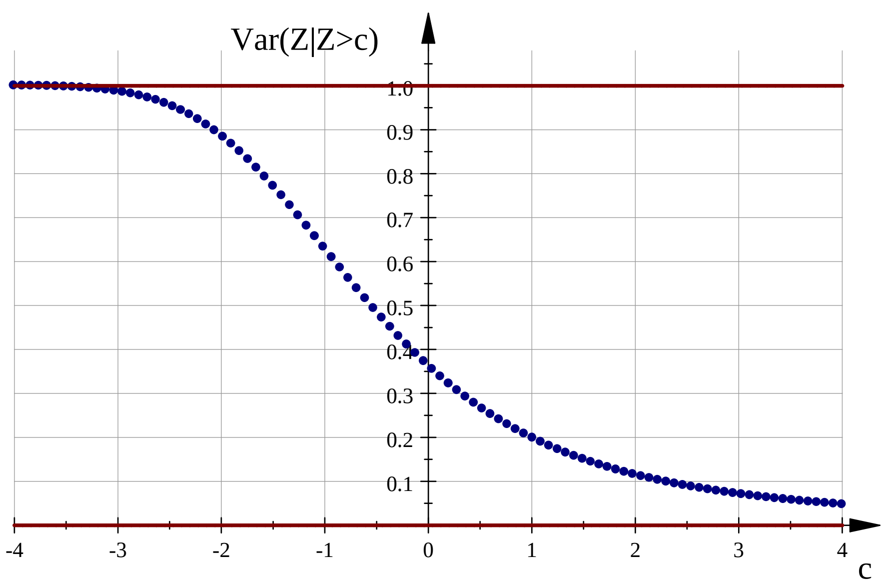
(P-11) $\underset{c\to-\infty}{Lim}\,Var(Z|Z>c) = 1$

Truncated Standard Normal Expectation



$E\left[Z|Z>c\right];\ Z\sim N\left(0,1\right)$

Truncated Standard Normal Variance



$Var\left(Z|Z>c\right);\ Z\sim N\left(0,1\right)$

2 More on Roy Model of Self-Selection

2.1 Model

Two-sector model[1]:

Agents are income maximizers, i.e., agent works in sector in which has highest income.

Mobility between sectors is costless, but they can work in only one sector (sector 1 or sector 2).

Each sector requires sector-specific task and agents have two skills T_1 and T_2.

Assume aggregate skill distribution given, i.e., short-run model. (No investment possibilities to change skills.)

Prices for skills are assumed to be known by agents at time of making sectoral choice decision. (Certainly of prices is not crucial.)

T_i denotes amount of sector i task an agent can perform.

π_i is price or return to worker for working in sector i and $\pi_i > 0$.

W_i denotes the wage in sector i where $W_i = \pi_i \cdot T_i$

Assume normality

Assume $(\ln T_1,\ \ln T_2)$ are normally distributed with mean (μ_1, μ_2) and covariance matrix $\sum$, i.e.,

$$\begin{pmatrix} \ln T_1 \\ \ln T_2 \end{pmatrix} \sim N\left(\begin{matrix} \mu_1 \\ \mu_2 \end{matrix}, \sum \right)$$

where $\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$. Define (u_1, u_2) as a mean zero normal vector, i.e.,

$$\begin{pmatrix} u_i \\ u_2 \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \sum \right)$$

As a consequence of these assumptions, we have:

$$\begin{aligned} \ln T_i &= \mu_i + u_i \\ W_i &= \pi_i \cdot T_i \Rightarrow \ln W_i = \ln \pi_i + \ln T_i \\ \text{then } \ln W_i &= \ln \pi_i + \mu_i + u_i \end{aligned}$$

Agent decision

The agent works in sector 1 iff:

[1] Results drawn on Heckman and Sedlace (*JPE*, 1985) and Heckman and Honoré (*E*, 1986)

$$W_1 = W_2$$

or

$$\begin{aligned} \ln W_1 &> \ln W_2 \\ \ln \pi_1 + \mu_1 + u_1 &> \ln \pi_2 + \mu_2 + u_2 \\ u_1 - u_2 &> \ln(\pi_2/\pi_1) + \mu_2 - \mu_1 \end{aligned}$$

So that proportion of population working in sector 1 given by proportion of population for which:

$$T_1 > \frac{\pi_2}{\pi_1} T_2$$

Then, it follows that

$$\Pr(i) = \Pr(\ln W_i > \ln W_j)$$

Now we want to prove 3 things

(1) $\Pr(i) = \Phi(c_i)$, then

(2) $E(\ln W_i | \ln W_i > \ln W_j) = \ln \pi_i + \mu_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*} \lambda(-c_i)$

(3) $Var(\ln W_i | \ln W_i > \ln W_j) = \sigma_{ii} \left\{ \rho_i^2 \left[1 - \lambda(c_i) c_i - \lambda^2(-c_i) \right] + \left(1 - \rho_i^2\right) \right\}$

where $i, j = 1, 2, i \neq j$, $c_i = c_i^*/\sigma^*, c_i^* = \ln(\pi_i/\pi_j) + \mu_i - \mu_j, \sigma^* = \sqrt{Var(u_1 - u_2)} = \sqrt{\sigma_{11} + \sigma_{22} - 2\sigma_{12}}$ and $\rho_i = Corr(u_i, u_i - u_j) = \frac{Cov(u_i, u_i - u_j)}{\sqrt{Var(u_i)}\sqrt{Var(u_i - u_j)}} = \frac{\sigma_{ii} - \sigma_{ij}}{(\sigma_{ii})^{1/2} \sigma^*}$.

For proof, define $D_i \equiv u_i - u_j$, then $u_i = a_i D_i + v_i$, where $a_i = \frac{Cov(D_i, u_i)}{Var(D_i)}, E(v_i) = 0, Var(v_i) = \sigma_{ii}(1 - \rho_i)$ and v_i is independent of D_i.

Proof of (1).

$$\begin{aligned} \Pr(i) &= \Pr(\ln W_i > \ln W_j) \\ &= \Pr\left(u_i - u_j > \ln(\pi_j/\pi_i) + \mu_j - \mu_i\right) \\ &= \Pr\left(\frac{u_i - u_j}{\sigma^*} > -\frac{c_i^*}{\sigma^*}\right) \\ &= \Pr(z > -c_i) \\ &= 1 - \Phi(-c_i) \\ &= \Phi(c_i) \end{aligned} \tag{1}$$

Proof of (2)

$$
\begin{aligned}
& E\left(\ln W_i \mid \ln W_i > \ln W_j\right) \\
= \; & E\left(\ln W_i = \ln \pi_i + \ln T_i \mid \ln \pi_i + \mu_i + u_i > \ln \pi_j + \mu_j + u_j\right) \\
= \; & \ln \pi_i + \mu_i + E\left(u_i | u_i - u_j > -c_i^*\right) \\
= \; & \ln \pi_i + \mu_i + E\left(a_i D_i + v_i | u_i - u_j > -c_i^*\right) \\
= \; & \ln \pi_i + \mu_i + \sigma^* a_i E\left(\frac{D_i}{\sigma^*} | \frac{D_i}{\sigma^*} > -\frac{c_i^*}{\sigma^*}\right) \\
= \; & \ln \pi_i + \mu_i + \sigma^* \frac{Cov\left(D_i, u_i\right)}{Var\left(D_i\right)} \lambda\left(-c_i\right) \\
= \; & \ln \pi_i + \mu_i + \frac{Cov\left(u_i - u_j, u_i\right)}{\sigma^*} \lambda\left(-c_i\right) \\
= \; & \ln \pi_i + \mu_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*} \lambda\left(-c_i\right) \qquad (2)
\end{aligned}
$$

By the same logic,

$$
\begin{aligned}
& E\left(\ln T_i \mid \ln W_i > \ln W_j\right) \qquad (3) \\
= \; & E\left(\mu_i + u_i \mid \ln W_i > \ln W_j\right) \\
= \; & \mu_i + E\left(u_i \mid \ln W_i > \ln W_j\right) \\
= \; & \mu_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*} \lambda\left(-c_i\right) \qquad (4)
\end{aligned}
$$

Proof of (3)

$$
\begin{aligned}
& Var\left(\ln W_i \mid \ln W_i > \ln W_j\right) \qquad (5) \\
= \; & Var\left(u_i | u_i - u_j > -c_i^*\right) \\
= \; & Var\left(a_i D_i + v_i | u_i - u_j > -c_i^*\right) \\
= \; & a_i^2 Var\left(D_i | D_i > -c_i^*\right) + Var\left(v_i\right) \\
= \; & a_i^2 \sigma^{*2} Var\left(\frac{D_i}{\sigma^*} | \frac{D_i}{\sigma^*} > -\frac{c_i^*}{\sigma^*}\right) + Var\left(v_i\right) \\
= \; & \left(a_i \sigma^*\right)^2 Var\left(z | z > -c_i\right) + Var\left(v_i\right), \text{ from (P-2)} \\
= \; & \sigma_{ii} \rho_i^2 \left(1 - \lambda\left(-c_i\right) c_i - \left(\lambda\left(-c_i\right)\right)^2\right) + \sigma_{ii}\left(1 - \rho_i^2\right) \\
= \; & \sigma_{ii}\left\{\rho_i^2\left[1 - \lambda\left(-c_i\right) c_i - \lambda^2\left(-c_i\right)\right] + \left(1 - \rho_i^2\right)\right\} \qquad (6) \\
= \; & \sigma_{ii}\left\{1 - \rho_i^2\left[\lambda^2\left(-c_i\right) - \lambda\left(-c_i\right)\left(-c_i\right)\right]\right\} \qquad (7)
\end{aligned}
$$

from (P-5) and $c = -c_i$, we have $0 < 1 - \rho_i^2\left[\lambda^2\left(-c_i\right) - \lambda\left(-c_i\right)\left(-c_i\right)\right] < 1$ in (7) easily. Consequently, (6) $\leq \sigma_{ii}$, i.e., $Var\left(\ln W_i \mid \ln W_i > \ln W_j\right) \leq \sigma_{ii} = Var\left(u_i\right) = Var\left(\ln W_i\right)$. Thus, sectoral variances always descrease with increased selection.

2.2 The Nature of Distribution of Skills and Earnings under Self-Selection

From (3), it follows that the mean observed level of log skills in a sector is given by

$$\begin{aligned} E(\ln T_1 | \ln W_1 > \ln W_2) &= \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\lambda(-c_1) \\ E(\ln T_2 | \ln W_2 > \ln W_1) &= \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*}\lambda(-c_2) \end{aligned}$$

then

$$E(\ln T_1 | \ln W_1 > \ln W_2) \begin{bmatrix} > \\ = \\ < \end{bmatrix} \mu_1 \text{ as } (\sigma_{11} - \sigma_{12}) \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0 \tag{8}$$

$$E(\ln T_2 | \ln W_2 > \ln W_1) \begin{bmatrix} > \\ = \\ < \end{bmatrix} \mu_2 \text{ as } (\sigma_{22} - \sigma_{12}) \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0 \tag{9}$$

For introduction to comparative and absolute advantage, we analysis 4 cases as followed first and others later.

(A) $\sigma_{11} - \sigma_{12} > 0, \sigma_{22} - \sigma_{12} > 0$

(B) $\sigma_{11} - \sigma_{12} > 0, \sigma_{22} - \sigma_{12} < 0$

(C) $\sigma_{11} - \sigma_{12} < 0, \sigma_{22} - \sigma_{12} > 0$

(D) $\sigma_{11} - \sigma_{12} < 0, \sigma_{22} - \sigma_{12} < 0$

Because of the positive definite of matrix $\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$, $\sigma_{11}\sigma_{22} - \sigma_{12}^2 > 0$, i.e., one of $(\sigma_{11} - \sigma_{12})$ and $(\sigma_{22} - \sigma_{12})$ must be positive. Then, case (D) is impossible.

(1) **Comparative Advantage**

If $\sigma_{11} - \sigma_{12} > 0$, self-selection always leads to the mean of $\ln T_1$ employed in sector 1 to exceed μ_1. At the same time, if $\sigma_{22} - \sigma_{12} > 0$ (Case A), then the mean of $\ln T_2$ employed sector 2 also exceeds μ_2. This is referred to as the case of **comparative advantage**, i.e., self-selection on income leads to workers sorting into sectors in which they have a comparative advantage in terms of their skills. Note that this case is more likely to occur when $\sigma_{12} < 0$, i.e., when the sector-specific skills are negatively correlated.

(2) **Absolute Advantage**

However, if $\sigma_{22} - \sigma_{12} < 0$ (Case B), self-selection always leads to mean of $\ln T_2$ employed in sector 2 to fall below μ_2. At the same time, the mean of $\ln T_1$ must lie above μ_1. Or, mean of $\ln T_1$ employed in sector 2 to fall below μ_1 and the mean of $\ln T_2$ must lie above μ_2 when $\sigma_{11} - \sigma_{12} < 0$ and $\sigma_{22} - \sigma_{12} > 0$ (Case C). Thus, this somewhat "unusual" case, i.e., that self-selection actually reduces the mean skills in the selected sector, can only occur in one sector, but not both. Note that these two cases require that σ_{12} be sufficiently positive. This is referred to as the case of **absolute advantage** or **hierarchical sorting**, i.e., agents tend to have high or low skills in both sectors.

For the other special cases

(3) If $\sigma_{12} = 0$, i.e., endowments of sector-specific skills are uncorrelated, self-selection always leads to mean of $\ln T_1$ employed in sector 1 to exceed μ_1 and to mean of $\ln T_2$ employed in sector 2 to exceed μ_2. We have the same situation with (1) —— Comparative advantage.

(4) If $\sigma_{11} = \sigma_{12}$, then there is no selection bias in sector 1, i.e., mean of $\ln T_1$ employed in sector 1 equals μ_1. Note that this is also the case where variance of $\ln T_1$ employed in sector 1 equal to the variance of $\ln T_1$ in the population.

(5) If $\sigma_{11} = \sigma_{12} = \sigma_{22}$, there is no selection bias in either sector. In this case, the sorting across sectors would look as if agents were randomly assigned to the two sectors.

Effects of Price Change on $\ln T_i$

The effects of an increase in $\ln \pi_1$ on mean skill in sector 1 and 2 are given by differentiating

$$E\left(\ln T_1 | \ln W_1 > \ln W_2\right) = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*} \lambda\left(-c_1\right)$$

and

$$E\left(\ln T_2 | \ln W_2 > \ln W_1\right) = \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*} \lambda\left(-c_2\right)$$

with respect to $\ln \pi_1$, i.e.,

$$\frac{\partial E\left(\ln T_1 | \ln W_1 > \ln W_2\right)}{\partial \ln \pi_1} = -\frac{\sigma_{11} - \sigma_{12}}{\left(\sigma^*\right)^2} \lambda'\left(-c_1\right) \quad (10)$$

$$\frac{\partial E\left(\ln T_2 | \ln W_2 > \ln W_1\right)}{\partial \ln \pi_1} = \frac{\sigma_{22} - \sigma_{12}}{\left(\sigma^*\right)^2} \lambda'\left(-c_2\right) \quad (11)$$

from (P-5), $\lambda'\left(-c_1\right) > 0$. Obviously, the (10) is negative if $\sigma_{11} - \sigma_{12} > 0$ and positive if $\sigma_{11} - \sigma_{12} < 0$. The (11), however, is opposite to (10) in the same conditions.

For (10),

$$\sigma_{11} - \sigma_{12} > 0 \Rightarrow \begin{cases} \sigma_{22} - \sigma_{12} > 0 \ \ (a) \Rightarrow \text{Comparative Advantage} \\ \sigma_{22} - \sigma_{12} < 0 \ \ (b) \Rightarrow \text{Absolute Advantage} \end{cases}$$

Intuitively, in (a) the correlation between two sectors is sufficiently low. This means that people who are good at sector 1 work in sector 1 already. If π_1 increase, more people but with worse skills will be drawn into sector 1, i.e., $E\left(\ln T_1 | \ln W_1 > \ln W_2\right)$ decreases. And in (b), correlation between sector 1 and 2 is positive and less than σ_{11}. The best who are good at sector 1 already work in sector 1, then the selection when π_1 increases will draw not so good people into sector 1, i.e., mean skills in sector 1 will fall.
In the same way,

$$\sigma_{11} - \sigma_{12} < 0 \Rightarrow \begin{cases} \sigma_{22} - \sigma_{12} > 0 \;\; (c) \Rightarrow \text{Absolute Advantage} \\ \sigma_{22} - \sigma_{12} < 0 \;\; (d) \Rightarrow \text{Impossibility} \end{cases}$$

so (d) is impossible. In (c), the correlation between two sectors is positive and greater than σ_{11}. The people with the worst skills in sector 1 already work in sector 1, then an increase in π_1 will draw people with better skills in sector 1 into sector 1.

For (11),

$$\sigma_{22} - \sigma_{12} > 0 \Rightarrow \begin{cases} \sigma_{11} - \sigma_{12} > 0 \;\; (a)' \Rightarrow \text{Comparative Advantage} \\ \sigma_{11} - \sigma_{12} < 0 \;\; (c)' \Rightarrow \text{Absolute Advantage} \end{cases}$$

This is similar to (a) and (c). Because the people with the best skills in sector 2 already work in sector 2, the worst working in sector 2 already will transfer from sector 2 to 1 as an increase in π_1, i.e., $E\left(\ln T_2 | \ln W_2 > \ln W_1\right)$ increases.
Similarity,

$$\sigma_{22} - \sigma_{12} < 0 \Rightarrow \begin{cases} \sigma_{11} - \sigma_{12} > 0 \;\; (b)' \Rightarrow \text{Absolute Advantage} \\ \sigma_{11} - \sigma_{12} < 0 \;\; (d)' \Rightarrow \text{Impossibility} \end{cases}$$

Here, the same situations with (b) and (d) happen. Since the mean skills of $\ln T_2$ employed in sector 2 are less than μ_1 from $\sigma_{22} - \sigma_{12} < 0$, the worst in sector 2 already work in sector 2, Then, the better working in sector 2 already will leave from sector 2, i.e., the $E\left(\ln T_2 | \ln W_2 > \ln W_1\right)$ decreases.

Effects of Price Change on $\ln W_i$

The discussion is like above. Here we just show one case as an example. An increase in π_1 reduces the average wage paid in both sectors.

Recall that

$$E\left(\ln W_1 | \ln W_1 > \ln W_2\right) = \ln \pi_1 + \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*} \lambda\left(-c_1\right)$$

and

$$\begin{aligned} E\left(\ln W_2 | \ln W_2 > \ln W_1\right) &= \ln \pi_2 + \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*} \lambda\left(-c_2\right) \\ &= \ln \pi_2 + \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*} \lambda\left(c_1\right) \end{aligned}$$

and differentiate them with respect to $\ln \pi_1$. For the negative effect of π_1 on W_1 and W_2, we get

$$\frac{\partial E\left(\ln W_1 | \ln W_1 > \ln W_2\right)}{\partial \ln \pi_1} = 1 - \frac{\sigma_{11} - \sigma_{12}}{\left(\sigma^*\right)^2} \lambda'\left(-c_1\right) < 0 \qquad (12)$$

$$\frac{\partial E\left(\ln W_2 | \ln W_2 > \ln W_1\right)}{\partial \ln \pi_1} = \frac{\sigma_{22} - \sigma_{12}}{\left(\sigma^*\right)^2} \lambda'\left(c_1\right) < 0 \qquad (13)$$

The following inequations are asked

$$\begin{aligned} \frac{\sigma_{11}-\sigma_{12}}{(\sigma^*)^2}\lambda'(-c_1) &> 1 \\ \frac{\sigma_{22}-\sigma_{12}}{(\sigma^*)^2}\lambda'(c_1) &< 0 \end{aligned}$$

then, from $\lambda'(\cdot) > 0$

$$\begin{aligned} \sigma_{11}-\sigma_{12} &> 0 \\ \sigma_{22}-\sigma_{12} &< 0 \end{aligned}$$

This is the Absolute advantage, and the best in sector 1 and 2 will work in sector 1. If π_1 increase, more people but with worse skill in sector 1 will joint to sector 1. And the people working in sector 2 is worst skill in sector 2. Then, there are the better skills who already working in sector 2 will leave.

$\sigma_{11}-\sigma_{12} > 0$ and $\sigma_{22}-\sigma_{12} < 0$ are the sufficient and necessary condition for (10) < 0 and (11) < 0 but not for (12) and (13). For (12) and (13) to occur, a necessary condition is that

$$\frac{\sigma_{11}-\sigma_{12}}{(\sigma^*)^2} = \frac{\sigma_{11}-\sigma_{12}}{(\sigma_{11}-2\sigma_{12}+\sigma_{22})} > 1$$

since $\lambda'(-c_1) \in (0,1)$ by (P-5). We can pick $\frac{\sigma_{11}-\sigma_{12}}{(\sigma_{11}-2\sigma_{12}+\sigma_{22})} > 1$ and make $\lambda'(-c_1)$ big enough. So that it is possible to have an increase in prices in one sector leading to a reduction in average wages paid in both sectors.

2.3 More on Effect of Self-Selection on Distribution of Earnings across sectors

Recall that in homework2 $X = (X_1, X_2)'$ have a joint Normal Distribution $N(\mu, \sum)$ where $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ and $\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$, then we have the nomal conditional distribution of X_i given $X_j = x_j$ as followed

$$N\left(\mu_i + \frac{\sigma_{ij}}{\sigma_{jj}}(x_j-\mu_j), \frac{\sigma_{ii}\sigma_{jj}-\sigma_{ij}^2}{\sigma_{jj}}\right)$$

where $i, j = 1, 2$ and $i \neq j$. In this model,

$$\begin{aligned} \ln T_1 &= \mu_1 + u_1 \\ \ln T_2 &= \mu_2 + u_2 \end{aligned}$$

and

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \sim \left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right)$$

Following the same logic, the regression equation for $\ln T_2$ condition on $\ln T_1$ is given by:

$$\ln T_2 = \mu_2 + \frac{\sigma_{12}}{\sigma_{11}} (\ln T_1 - \mu_1) + \varepsilon_2 \tag{14}$$

where $\varepsilon_2 \sim N\left(0, \frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{11}}\right)$ and define $\rho \equiv corr\,(u_1, u_2) = \frac{\sigma_{12}}{\sqrt{\sigma_{11}\sigma_{12}}}$.

Now, to gain further insight into the effect of self-selection on the distribution of earnings, consider the following three cases:

Case 1: $\sigma_{12} = \sigma_{11}$

Case 2: $\sigma_{12} > \sigma_{11}$

Case 3: $\sigma_{12} < \sigma_{11}$

Case 1 *Figure 1 show the case 1 added* $\mu_2 > \mu_1 > 0$ *and* $\sigma_{12} > 0$.

Points to note:

(a) Agens with endowments of $(\ln T_1, \ln T_2)$ above the 45^{o} line (equal income line) choose to work in sector 2 and those below choose to work in sector 1.

(b) For any given value of $\ln T_1 = \ln t_k$, the same proportion of agents work in sector 2, for all k.Therefore, the distribution of $\ln T_1$ employed in sector 2 is the same as in the latent population distribution, i.e., there is no selection bias in sector 2, and mean of $\ln T_1$ employed in sector 2 equals μ_1. Furthermore, note that it is also the case that the variance of $\ln T_1$ employed in sector 2 is equal to the variance of $\ln T_1$ in the population. Finally, note that if $\sigma_{12} = \sigma_{11} = \sigma_{22}$, there is no selection bias in either sector. In this case, the sorting across sectors would look as if agents were randomly assigned to the two sectors.

(c) It follows from (b) that if raise π_1 (or lower π_2), which shifts the 45^{o} line upward, the same proportion of people enter sector 1 at each value of $\ln T_1 = \ln t_k$ for all k.

Case 2 *Now consider Figure 2 which plots* (14) *when* $\sigma_{12} > \sigma_{11}$ *and* $\mu_2 > \mu_1 > 0$.

Points to note:

(a) As we have already seen for this case, the mean of skill level in sector 1 is lower than the population mean level of $\ln T_1$.

(b) Moreover, agents with high amounts of $\ln T_1$ are under-represented in sector 1 because $\mu_2 > \mu_1$.

(c) Note that in the extreme case, where $\ln T_1$ and $\ln T_2$ are perfectly positively correlated, we have the extreme version of absolute advantage or hierarchical sorting. In this case, the hightest paid worker in sector 1 earns the same as the lowest paid worker in sector 2

(d) Now if we raise π_1 (or lower π_2), attracting workers to sector 1, the mean of $\ln T_1$ must go up, but it will take the workers from the upper end of the $\ln T_1$ distibution. Accordingly, an $x\%$ increase in π_1 leads to a more than $x\%$ increase average $\ln T_1$ and thus $\ln W_1$ ($\ln W_1 = \ln \pi_1 + \ln T_1$) in sector 1.

Case 3 *Finally, if we consider $\sigma_{12} < \sigma_{11}$ and $\mu_2 > \mu_1 > 0$, then:*

(a) Again, as we have already seen, mean of $\ln T_1$ will exceed μ_1 in equilibrium.

(b) Moreover, the proportion of workers from each $\ln T_1 = \ln t_k$ group working in this sector will increase with higher values of $\ln T_1$.

(c) Since comparative advantage is playing out in this case, an $x\%$ increase in π_1 leads to a less than $x\%$ increase average $\ln T_1$ and thus $\ln W_1$ in sector 1.

(d) Note that it is possible that if $\sigma_{12} > \sigma_{22}$ an increase in π_1 can cause measured sector 1 wages to decline.

Figure 1-3

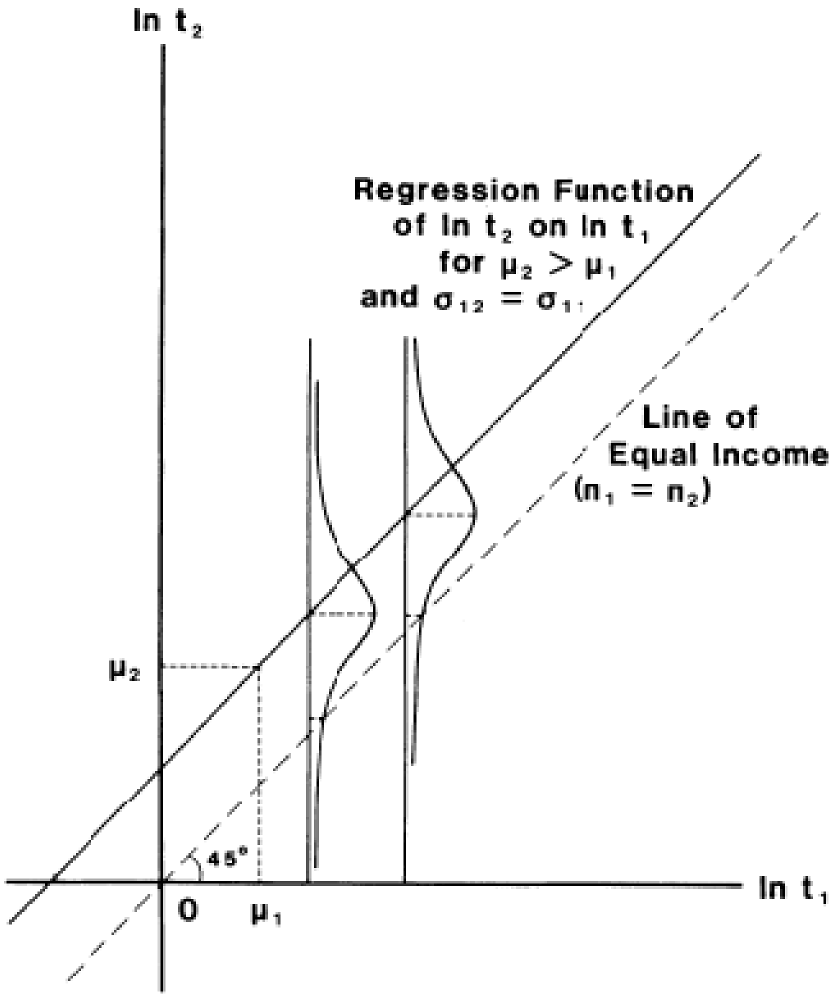


Figure 1

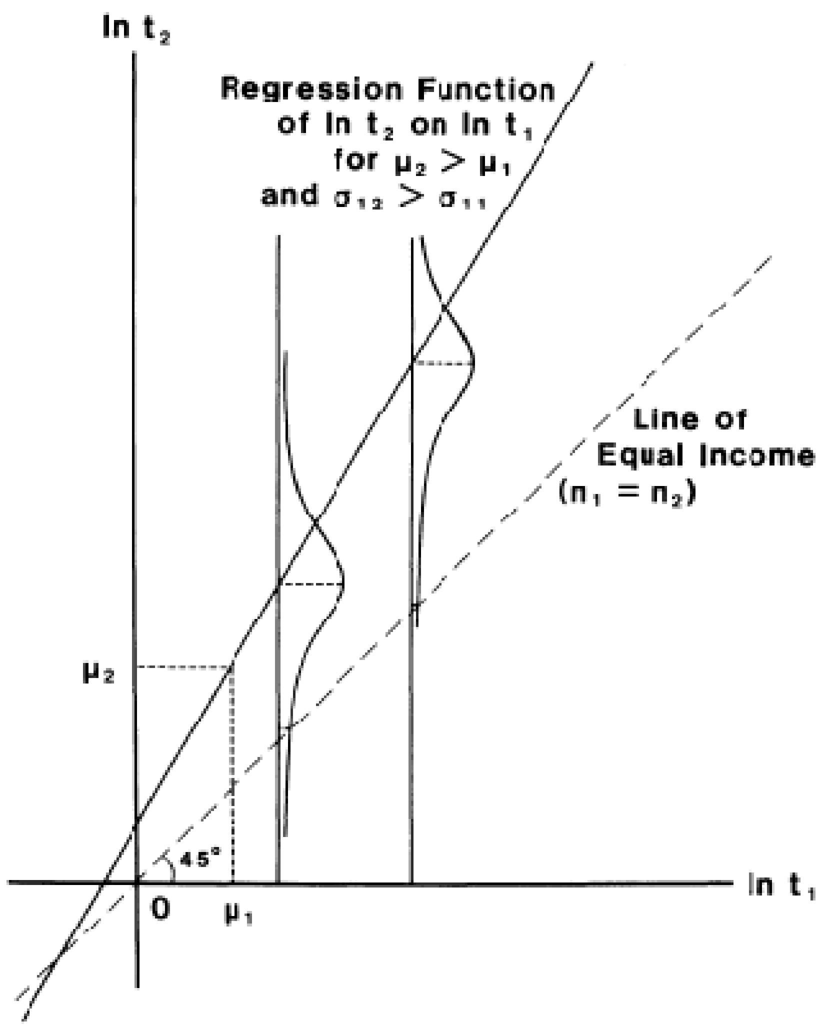

ln t_2
Regression Function of ln t_2 on ln t_1 for $\mu_2 > \mu_1$ and $\sigma_{12} > \sigma_{11}$
Line of Equal Income ($n_1 = n_2$)
μ_2
45°
0
μ_1
ln t_1

Figure 2

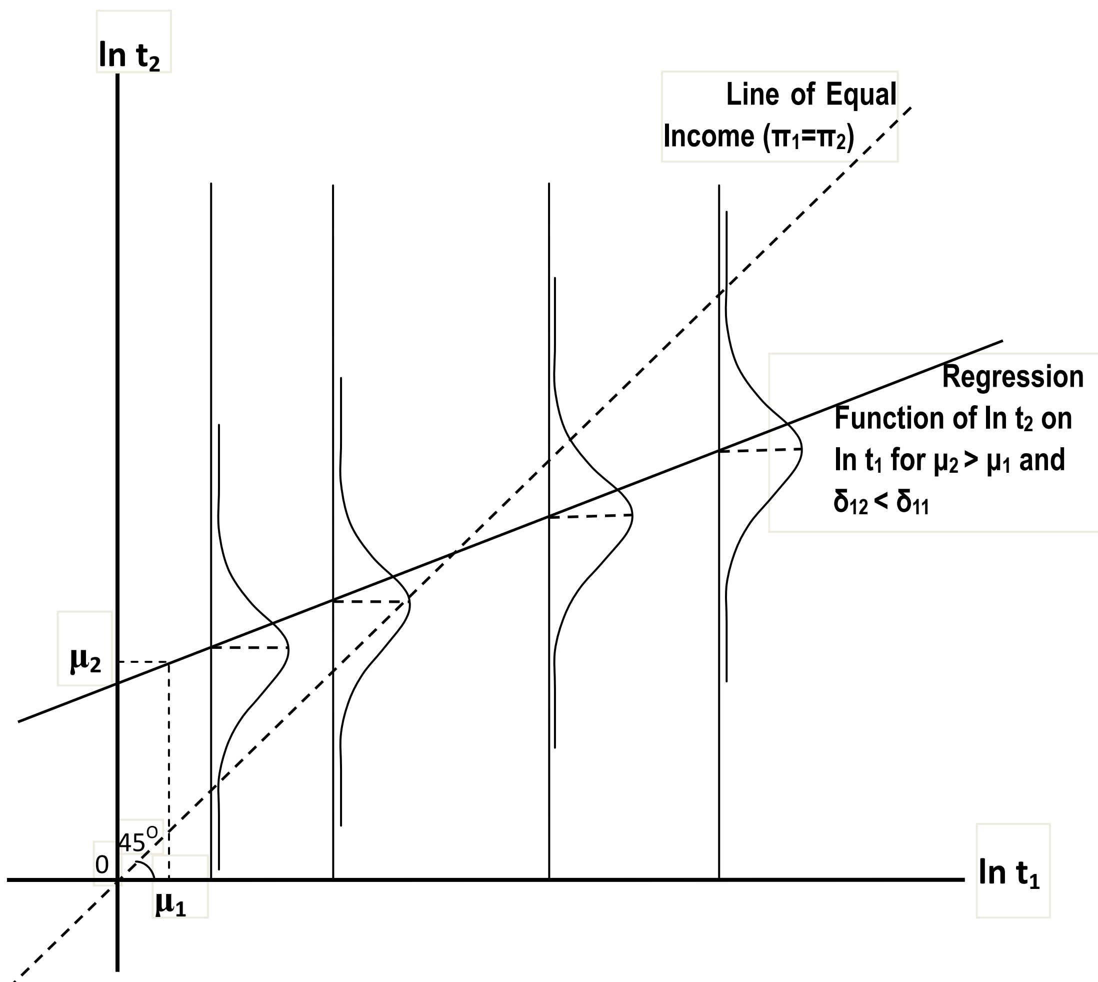

ln t_2
Line of Equal Income (π_1=π_2)
Regression Function of ln t_2 on ln t_1 for $\mu_2 > \mu_1$ and $\delta_{12} < \delta_{11}$
μ_2
45°
0
μ_1
ln t_1

Figure 3