Lecture 1: The Roy Model of Self-Selection: Simple Case

A core topic in labor economics is 'self-selection.' What this means in theory is that rational actors make optimizion decisions about what markets to participate in — job, location, education, marriage, crime, etc. What it means in practice is that observed economic relationship should generally be views as endogenous outcomes of numerous optimizing decisions, rather than as exogenous causal relationships. Understanding self-selection should make you skeptical of treading any ecological correlation as causal.

The starting point of formal treatmeat of this topic in economics is Roy's(1951) "Thoughts on the Distribution of Earnings," which discusses the optimizing is that there are three factors that affects this choice:
1. Fundamental distribution of skills and abilities
2. The correlations among these skills in the population
3. The technologies for applying these skills
4. Consumer tastes that impact demand for different types of outputs

At the time of Roy's writing, the presumption was that the distribution of income that arises from economic processes is arbitrary. Hence, if we compare the mean earnings if hunters and fishemen, $\overline{y}_h$ and $\overline{y}_f$, then $\overline{y}_h - \overline{y}_f$ is an estimate of the earnings gain or less that an individual would receive from switching from fishing to hunting. Roy's article explains why this view is incorrect.

The essential departure of Roy's model from previous work is that it is a multiple-index model (in this case, 2 indices); workers have skills in each occupation, but they can only use one skill or the other. Hence, workers self-select the sector that gives them the highest expected earnings. Equilibrium in each market equates supply and demand, while a self-selection condition means that the marginal worker is indifferent between the two sectors.

In these notes, I lay out the simpliest version of the Roy Model. To do so, I follow the structure of the paper by George Borjas, "Self-Selection and the Earnings of Imnigrants" in the *AER*, 1987. This paper characterizes a simple, parametric 2-sector Roy model. The enduring contribution of Borjas' paper is this model rather than the empirical findings. Any labor economist should be well versed with this model.

1 Borjas 1987: Self-Selection and the Earnings of Immigrants

Who chooses to immigrante to the United States? One ready-made answer is that workers from low wage countries will immigrate. This may be true on average, but it's probably too simple. The workers immigrating to the United States are probably not a random subset of the Mexican workforce. Rather, we should expect that potential migrants make some rough comparison of their wages in the home country and their expected wages in the U.S. On average, we should expect those who immigrate to have higher expected earnings in the U.S. than Mexico and vice versa for those who stay.

Setup

♦ Consider two countries Mexico (0) and the United States (1), denoting the "source" and "host" country, respectively.

♦ Log earnings in the source country are given by

$$w_0 = \mu_0 + \varepsilon_0 \tag{1}$$

where $\varepsilon_0 \sim N\left(0, \sigma_0^2\right)$. It is useful to think of ε_0 as the de-meaned value of worker's 'skill' in Mexico (the source country).

⧫ If everyone from Mexico were to migrate to the U.S., their earnings would be

$$w_0 = \mu_0 + \varepsilon_0 \tag{2}$$

where $\varepsilon_1 \sim N\left(0, \sigma_1^2\right)$.
⧫ Assume that the cost of migrating is π.
⧫ Assume further that each worker knows C, μ_0,μ_1 and his individual epsilons: ε_0 and ε_1. Yet, economists only observe ε_0 or ε_1 for any individual.

Self-Selection Decision Rule:
A Mexican worker will choose to migrate to the U.S. iff

$$(\mu_1 - \mu_0 - \pi) + (\varepsilon_1 + \varepsilon_0) > 0 \tag{3}$$

Define the indicator variable $I = 1$, *if this selection condition is satisfied and* $I = 0$, *otherwise.*

⧫ Define $v \equiv \varepsilon_1 + \varepsilon_0$, The probability that a randomly chosen worker from Mexico choose to migrate to the U.S. is equal to

$$\begin{aligned} P &= \Pr[I = 1] \\ &= \Pr[v > \mu_0 - \mu_1 + \pi] \\ &= \Pr\left[\frac{v}{\sigma_v} > \frac{\mu_0 - \mu_1 + \pi}{\sigma_v}\right] \\ &= 1 - \Phi\left(\frac{\mu_0 - \mu_1 + \pi}{\sigma_v}\right) \\ &= 1 - \Phi(z) \end{aligned} \tag{4}$$

where $z = \frac{\mu_0 - \mu_1 + \pi}{\sigma_v}$ and $\Phi(\cdot)$ is the CDF of the standard normal. Notice that the higher is z, the lower is the probability of migration (from Mexico to the U.S.). This is because z is rising in the mean earnings of Mexico and the cost of migration. So it follows that

$$\frac{\partial P}{\partial \mu_0} < 0, \frac{\partial P}{\partial \mu_1} > 0, \frac{\partial P}{\partial \pi} < 0.$$

⧫ These are the **mean effects** and drive, in part, self-selection of agents. In our example, high mean wages in the U.S. relative to those in Mexico, create a net incentive for workers to migrate from Mexico to the U.S. Variation in the "other" costs of migration also create such incentives.
⧫ However, there is more to the nature of self-selection in the Roy Model. To see these Implications of this model, it is useful to assume from here forward that

$$\mu_0 \approx \mu_1 \tag{5}$$

so we can focus on self-selection properties of the simple Roy Model rather than its implications for mean differences.

1.1 Selection Conditions

For figuring out the conditions, we need some mathematics to support.
Supplement for Mathematics

Proposition 1 *for any two normal variables* (u_1, u_2), *we can write* $u_2 = \alpha_0 + \alpha_1 u_1 + v$, *where* v *has a normal distribution with mean zero and* v *is independent of* u_1. *Then we can get*

$$\alpha_1 = \frac{Cov(u_1, u_2)}{Var(u_1)}; \alpha_0 = E(u_2) - E(u_1) \tag{6}$$

Proposition 2 ***Inverse Mill's Ratio:*** *if we have $z \sim N(0,1)$ and c is a scalar, then*

$$E(z|z>c) = \frac{\phi(c)}{\Phi(-c)} \equiv \lambda(z) \tag{7}$$

Now we calculate the $E(w_0|I=1)$ and $E(w_1|I=1)$ and get selection conditions.

$$\begin{aligned} E(w_0|I=1) &= E(\mu_0 + \varepsilon_0|I=1) \\ &= \mu_0 + E\left(\varepsilon_0|\frac{v}{\sigma_v} > z\right) \\ &= \mu_0 + \sigma_0 E\left(\frac{\varepsilon_0}{\sigma_0}|\frac{v}{\sigma_v} > z\right) \\ &= \mu_0 + \sigma_0 E\left(\frac{\frac{\sigma_{0V}}{\sigma_V^2}v + \xi}{\sigma_0}|\frac{v}{\sigma_v} > z\right) \\ &= \mu_0 + \sigma_0\rho_{0v} E\left(\frac{v}{\sigma_v}|\frac{v}{\sigma_v} > z\right) \\ &= \mu_0 + \sigma_0\rho_{0v}\left(\frac{\phi(z)}{\Phi(-z)}\right) = \mu_0 + \sigma_0\rho_{0v}\lambda(z) \end{aligned}$$

where $\rho_{0v} \equiv \frac{Cov(\varepsilon_0, v)}{\sigma_0\sigma_v} = \frac{\sigma_{01}-\sigma_0^2}{\sigma_0\sigma_v}$, so

$$E(w_0|I=1) = \mu_0 + \frac{\sigma_0\sigma_1}{\sigma_v}\left(\rho - \frac{\sigma_0}{\sigma_1}\right)\lambda(z) \tag{8}$$

where $\rho = \frac{\sigma_{01}}{\sigma_0\sigma_1}$. For similar reason,

$$\begin{aligned} E(w_1|I=1) &= \mu_1 + \sigma_1\rho_{1v}\left(\frac{\phi(z)}{\Phi(-z)}\right) \\ &= \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_v}\left(\frac{\sigma_1}{\sigma_0} - \rho\right)\lambda(z) \end{aligned} \tag{9}$$

1.2 Ways in Which Self-Selection Alter Expected Wages

Define $Q_0 \equiv E(w_0|I=1) - E(w_0) = E(\varepsilon_0|I=1)$ and $Q_1 \equiv E(w_1|I=1) - E(w_1) = E(\varepsilon_1|I=1)$. where Q_0 and Q_1 are the truncated means of the unobserved components of earnings in Mexico and earnings in the U.S., given migration from Mexico to the U.S..
Now we wil get four different cases easily.
Case 1: $Q_0 > 0, Q_1 > 0$
Case 2: $Q_0 < 0, Q_1 < 0$
Case 3: $Q_0 < 0, Q_1 > 0$
Case 4: $Q_0 > 0, Q_1 < 0$

Case 1 ***Positive Selection:*** *$Q_0 > 0$ and $Q_1 > 0$. The necessary and sufficient conditions for positive selection to occur are*

$$\frac{\sigma_1}{\sigma_0} > 1 \tag{a}$$

$$\rho > \min\left(\frac{\sigma_0}{\sigma_1}, \frac{\sigma_1}{\sigma_0}\right) \tag{b}$$

(a) implies that U.S. has a more diversed income than does Mexico. In effect, this condition implies that the U.S. has a higher "return to skill" than does Mexico.
(b) implies that the correlation between the skills valued in U.S. Mexico is sufficiently high, If you were a skiied worker in Mexico, you would not want to migrate to the U.S. a host country with a very high return to skills if the skills valued in the U.S. were uncorrelated with skills value in Mexico.

Case 2 ***Negative Selection:*** *$Q_0 < 0$ and $Q_1 < 0$. The necessary and sufficient conditions for negative selection to occur are*

$$\frac{\sigma_1}{\sigma_0} < 1, \rho > \min\left(\frac{\sigma_0}{\sigma_1}, \frac{\sigma_1}{\sigma_0}\right)$$

Case 3 ***Refugee Sorting:*** *$Q_0 < 0$ and $Q_1 > 0$. The necessary and sufficient conditions for negative selection to occur are*

$$\rho < \min\left(\frac{\sigma_0}{\sigma_1}, \frac{\sigma_1}{\sigma_0}\right)$$

Case 4 ***Impossibility:*** *$Q_0 > 0$ and $Q_1 < 0$. The necessary and sufficient conditions for negative selection to occur are*

$$\rho > \max\left(\frac{\sigma_0}{\sigma_1}, \frac{\sigma_1}{\sigma_0}\right)$$

Obviously, this case will never happen.

2 Heckman and Sedlacek 1985: Heterogeneity, Aggregation and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market

Set up
T_i denotes amount of task an agent can perform in sector i.
π_i is the price to worker if they work in sector i.
Assume

$$\ln T_i = \mu_i + u_i$$

and

$$\begin{pmatrix} \ln T_1 \\ \ln T_2 \end{pmatrix} \sim \left(\begin{matrix} u_1 \\ u_2 \end{matrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right)$$

Then, the totol earnings w_i for an agent is

$$w_i = \pi_i \cdot T_i \Rightarrow \ln w_i = \ln \pi_i + \ln T_i$$
$$\Longrightarrow \begin{cases} \ln w_1 = \ln \pi_1 + \mu_1 + u_1 \\ \ln w_2 = \ln \pi_2 + \mu_2 + u_2 \end{cases}$$

The agent works in sector i, iff:

$$w_i = \pi_i \cdot T_i > w_j = \pi_j \cdot T_j$$

or

$$\begin{aligned} &\ln w_i > \ln w_j \\ &\quad \Longrightarrow \ln \pi_i + \mu_i + u_i > \ln \pi_j + \mu_j + u_j \\ &\quad \Longrightarrow u_i - u_j > \ln\left(\frac{\pi_j}{\pi_i}\right) + \mu_j - \mu_i \end{aligned}$$

So, the probability of people work in sector i will be

$$\begin{aligned}
&\Pr\left(\ln w_i > \ln w_j\right) \\
&= \Pr\left(u_i - u_j > \ln\pi_j - \ln\pi_i + \mu_j - \mu_i\right) \\
&= \Pr\left(D_i > -c_i^*\right) \\
&= \Pr\left(\frac{D_i}{\sigma^*} > \frac{-c_i^*}{\sigma^*}\right) \\
&= \Pr\left(z > -c_i\right) = 1 - \Phi\left(-c_i\right) = \Phi\left(c_i\right) \qquad (10)
\end{aligned}$$

where $D_i = u_i - u_j,\ c_i^* = \ln\pi_i - \ln\pi_j + \mu_i - \mu_j,\ \sigma^* = \sqrt{Var\left(u_i - u_j\right)} = \sqrt{\sigma_{11} + \sigma_{22} - 2\sigma_{12}}$ and $z = \frac{D_i}{\sigma^*}$, $c_i = \frac{c_i^*}{\sigma^*}$. Equation (10) is based on the distribution of z.
Now we proof the following equation

$$E\left(\ln w_i | \ln w_i > \ln w_j\right) = E\left(\ln\pi_i + u_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*}\lambda\left(c_i\right)\right) \qquad (11)$$

where $\lambda\left(-c_i\right) = \frac{\left(\sqrt{2\pi}\right)^{-1}\exp\left(-\frac{1}{2}c_i^2\right)}{\Phi\left(-c_i\right)}$.
Proof.

$$\begin{aligned}
&E\left(\ln w_i | \ln w_i > \ln w_j\right) \\
&= E\left(\ln\pi_i + \mu_i + u_i | \ln\pi_i + \mu_i + u_i > \ln\pi_j + \mu_j + u_j\right) \\
&= \ln\pi_i + \mu_i + E\left(u_i | u_i - u_j > -[\ln\left(\frac{\pi_i}{\pi_j}\right) + \mu_i - \mu_j]\right) \\
&= \ln\pi_i + \mu_i + E\left(\frac{Cov\left(D_i, \mu_i\right)}{VarD_i} D_i | D_i > -c_i^*\right) \\
&= \ln\pi_i + \mu_i + \sigma^* \frac{Cov\left(D_i, u_i\right)}{VarD_i} E\left(\frac{D_i}{\sigma^*} | \frac{D_i}{\sigma^*} > -\frac{c_i^*}{\sigma^*}\right) \\
&= \ln\pi_i + \mu_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*} E\left(z | z > -c_i\right) \\
&= \ln\pi_i + \mu_i + \frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*}\lambda\left(-c_i\right)
\end{aligned}$$

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Then comparing (11) with $E\left(\ln w_i\right) = \ln\pi_i + \mu_i$, we can also analyse the differential $\frac{\sigma_{ii} - \sigma_{ij}}{\sigma^*}\lambda\left(-c_i\right)$ like what Borjas (1987) did.

For similar reason, the dissimilitude between $Var\left(\ln w_i | \ln w_i > \ln w_j\right)$ and $Var\left(\ln w_i\right)$ should be done.